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Eigenstructure Assignment Approach for Structural Damage Detection

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In this work, a methodology for incorporating measured modal data into an existing refined finite element model is examined with the objective of detecting and locating structural damage. The algorithm is based on the partial inverse problem, in that only partial spectral information is required. The technique utilizes a symmetric eigenstructure assignment algorithm to perform the partial spectral assignment. Algorithms to enhance mode shape assignability and to preserve sparsity in the damaged FEM are developed. The sparsity preservation is of particular importance when considering damage detection in truss-like structures. Several examples are presented to highlight the key points made within the paper.

I. Introduction

THE advent of the Space Shuttle has prompted considerable attention to the design and control of large space structures. Due to the large size and complexity of envisioned structures, as well as the use of advanced materials to reduce structural weight, it may become necessary to develop a structural health monitoring system to detect and locate structural damage as it occurs. From experience gained in the machinery health monitoring (diagnostic) field, one would expect the vibration signature of the structure (either frequency response functions and/or modal parameters) to provide useful information for locating damage in the structure.

Assume that a refined finite element model (FEM) of the structure has been developed before damage has occurred. By refined, we mean that the measured and analytical modal properties are in agreement. Next, assume at some later date that some form of structural damage has occurred. If significant, the damage will result in a change in the structures modal parameters. The question is: can the discrepancy between the original FEM modal properties and post-damage modal properties be used to locate and determine the extent of structural damage?

Some prior work in damage detection has used the general framework of FEM refinement (system identification) in the development of damage location algorithms. The motivation behind the development of FEM refinement techniques is based on the need to "validate" engineering FEMs before their acceptance as the basis for final design analysis. In this problem, the discrepancy between FEM modal parameters and experimental measurements is due to errors in both the original FEM and fabrication-induced errors. The standard problem has been to seek a refined FEM that is as close to (usually measured by some matrix norm) the original FEM and whose modal properties are in agreement with those that are measured subject to various constraints, such as symmetry and sparsity preservation.

The majority of algorithms used to address the FEM refinement problem can be broadly classified among three different classes. The first, and to date arguably the largest class of FEM refinement algorithms, is that of optimal matrix updates. Early work in optimal matrix updates using measured test data was performed by Rodden, who used ground vibration test data to determine the structural influence coefficients of a structure.[1] Brock examined the problem of determining a matrix that satisfied a set of measurements as well as enforcing symmetry and positive definiteness.[2] Berman and Flannelly discussed the calculation of property matrices when the number of measured modes is not equal to the number of degrees of freedom (DOF) of the FEM.[3]

Several optimal matrix update algorithms are based on the problem formulation set forth by Baruch and Bar Itzhack.[4] In their work, a closed-form solution was developed for the minimal Frobenius-norm matrix adjustment to the structural stiffness matrix incorporating measured frequencies and mode shapes. Berman and Nagy adopted a similar formulation but included approaches to improve both the mass and stiffness matrices.[5] In the previously cited work,[1-5] the zero/nonzero (sparsity) pattern of the original stiffness matrix may be destroyed. Algorithms by Kabe,[6] Kammer,[7] and Smith and Beattie[8] have been developed which preserve the original stiffness matrix sparsity pattern, thereby preserving the original load paths of the structural model. The Kabe[6] algorithm utilizes a percentage change in stiffness value cost function and appends the sparsity pattern as an additional constraint; whereas Kammer[7] and Smith and Beattie[8] investigate alternate matrix minimization formulations. Smith and Hendricks have utilized these various matrix updates in direct studies of damage location in large truss structures.[9,10] Although minimization of the matrix norm of the difference between the original and refined stiffness matrix is justified for the model refinement case, its applicability for damage detection is open to question because damage typically results in localized changes in the property matrices; whereas, the matrix norm minimization would tend to "smear" the changes throughout the entire stiffness matrix.

Sensitivity methods for model refinement/damage detection make use of sensitivity derivatives of modal parameters with respect to physical design variables.[11-13] The derivatives are then used to update the physical parameters. These algorithms result in updated models consistent within the original finite element program framework. Direct application of nonlinear optimization to the damage detection problem has been studied by Hajela and Soeiro[14] and Soeiro.[15] In this technique, it is required that the physical design variables be chosen such that the properties of the damaged component can be varied. This presents a practical difficulty in that the number of design variables required may grow quite large, although techniques utilizing continuum approximations are discussed as one possible solution.

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Control-based eigenstructure assignment techniques determine the pseudo-control that would be required to produce the measured modal properties with the initial structural model.[16–18] The pseudo-control is then translated into matrix adjustments applied to the initial FEM. In Ref. 16, two techniques are discussed for FEM refinement. The first assigns both eigenvalue and eigenvector information to produce updated damping and stiffness matrices. An unconstrained numerical nonlinear optimization problem is posed to enforce symmetry of the resulting model. A second approach, in which only eigenvalue information is used, uses a state-space formulation that finds the state matrix that has the measured eigenvalues and that is closest to the original state matrix. In Refs. 17 and 18, eigenvalue and eigenvector information is incorporated in the FEM using a symmetry preserving eigenstructure assignment theorem. This algorithm replaces the unconstrained optimization approach of Ref. 16 with the solution of a generalized algebraic Riccati equation whose dimension is defined solely by the number of measured modes. It should be noted that both the sensitivity and eigenstructure assignment algorithms, which do not demand the matrix norm minimization, may prove quite suitable for damage detection.

Notable exceptions to the direct use of FEM refinement algorithms to the damage detection problem are the work of Refs. 19, 20, and 21. In the work of Ref. 19, the flexibility matrix is determined using experimental data. This matrix is then multiplied by the original stiffness matrix, with those rows and/or columns that differ significantly from a row and/or column of the identity matrix indicating which degrees of freedom have been most affected by the damage. It is then assumed that damage has occurred in structural elements connecting those degrees of freedom. Although this algorithm provides information concerning location of damage, it is difficult to determine the extent of damage. In the work of Refs. 20 and 21, damage indicators and scales are discussed within the context of expert systems as an engineering tool for locating damage in civil engineering structures.

In the work presented in this paper, the eigenstructure assignment algorithm of Ref. 17 is extended to approach the damage location problem better. A subspace rotation algorithm is developed to enhance eigenvector assignability. Because load path preservation may be important in certain classes of damage detection, an iterative algorithm is presented that preserves the load path if the experimental data is consistent. The algorithm is tested and compared to other techniques on both simulated and actual experimental data.

II. Problem Definition

Consider a n-DOF structural model with feedback control

$$M\ddot{w} + D\dot{w} + Kw = B_o u \tag{1}$$

where M, D, and K are the $n \times n$ analytical mass, damping, and stiffness matrices, w is a $n \times 1$ vector of positions, B_o is the $n \times m$ actuator influence matrix, u is the $m \times 1$ vector of control forces, and the overdots represent differentiation with respect to time.

The results of an experimental modal test of the structure are assumed available, and that p eigenvalues λ_i and eigenvectors $\tilde{v}_i$ have been identified, $p < n$. The identified eigenvectors $\tilde{v}_i$ are of dimension $n \times 1$. Only s, $s < n$, components of $\tilde{v}_i$ have been actually measured. The unmeasured components of $\tilde{v}_i$ are set to zero.

In addition, the $r \times 1$ output vector y of sensor measurements is given by

$$y = C_o w + C_1 \dot{w} \tag{2}$$

where C_0 and C_1 are the $r \times n$ output influence matrices. The control law is taken to be a general linear output feedback controller

$$u = Fy \tag{3}$$

where F is the $m \times r$ feedback gain matrix. It is proven that if a system described by Eqs. (1) and (2) are controllable and observable, then by proper selection of F, max(m,r) closed-loop (controlled) eigenvalues can be assigned, max(m,r) closed-loop eigenvectors can be partially assigned with min(m,r) entries in each eigenvector being arbitrarily assigned.[22]

Substituting Eqs. (3) and (2) into (1), and rearranging

$$M\ddot{w} + (D - B_0FC_1)\dot{w} + (K - B_0FC_0)w$$
$$= M\ddot{w} + D_a\dot{w} + K_a w = 0 \tag{4}$$

one can see that the matrix triple products B_0FC_0 and B_0FC_1 result in changes in the stiffness and damping matrices respectively. These triple products can then be viewed as perturbation matrices to the stiffness and damping matrices such that the adjusted finite element model matches closely the experimentally measured modal properties. Unfortunately, these perturbation matrices are, in general, nonsymmetric when calculated using standard eigenstructure assignment techniques, thus yielding adjusted stiffness and damping matrices that are also nonsymmetric.

III. Problem Formulation

A. Eigenstructure Assignment Refinement Algorithm

1. *Introduction*

In this technique, the use of a symmetric eigenstructure assignment algorithm is investigated to determine the refined finite element model of the damaged structure by assigning the partial spectral measurements. The direct improvement algorithm determines residual damping and stiffness matrices such that the improved analytical model eigenstructure (or equivalently natural frequencies, damping ratios and mode shapes) matches more closely that obtained experimentally. The damping model may be nonproportional. Post-inspection of the changes made to the stiffness and damping matrices can then be used to determine the location and extent of structural damage.

2. *Symmetric Eigenstructure Assignment*

In the symmetric eigenstructure assignment algorithm, the feedback gain matrix F of Eq. (3), is calculated using standard eigenstructure assignment theory such that the adjusted finite element model matches the experimental modal data.[22] An arbitrary selection of B_0, C_0, and C_1 would result in nonsymmetric perturbation matrices. For the perturbations to be symmetric, the following conditions must be met

$$B_0FC_i = C_i^TF^TB_0^T, \qquad i = 0, 1 \tag{5}$$

The following two assumptions are now made. First, the number of pseudo-sensors and pesudo-actuators are taken to be equal to twice the number of measured modes ($m = r = 2p$). This is a requirement such that the inverse of certain matrices exist. Second, the C_i are written as

$$C_i = G_iB_0^T, \qquad i = 0, 1 \tag{6}$$

where G_i are $m \times m$ matrices and G_i^{-1} exists. Substituting Eq. (6) into (5), the symmetry statement can be written as:

$$FG_i = G_i^TF^T = G_i^*F^*, \qquad i = 0, 1 \tag{7}$$

where the superscript ( )* is the complex conjugate transpose operator.

A necessary but not sufficient condition on the G_i for symmetric perturbation matrices can be expressed in terms of a generalized algebraic Ricatti equation[17]

$$A_1X + XA_2 + XA_3X + A_4 = 0$$

$$X = G_1^{-1}G_0$$

$$A_1 = [\sigma^*\alpha^{-1}\sigma^{-1}(\alpha^*\tau^* - \tau\alpha) - \tau^*]\sigma^{-*}$$

$$A_2 = \tau^*\alpha^{-1}\sigma^{-1}\alpha^*$$

$$A_3 = \tau^*\alpha^{-1}\sigma^{-1}(\alpha^*\tau^* - \tau\alpha)\sigma^{-*}$$

$$A_4 = \sigma^*\alpha^{-1}\sigma^{-1}\alpha^* - I_m$$

$$\tau = \begin{bmatrix} W^*B_0 \\ \overline{W}^*B_0 \end{bmatrix} \quad \sigma = \begin{bmatrix} \Lambda W^*B_0 \\ \overline{\Lambda W}^*B_0 \end{bmatrix} \quad \alpha = Z - A_LV \tag{8}$$

where W is a matrix whose columns consist of the expanded achievable eigenvectors, Λ is a diagonal matrix of measured eigenvalues, the overbar is the complex conjugate operator, the superscript $(\ )^{-*}$ indicates the inverse of the complex conjugate transpose matrix, and Z, A_L, and V are matrices defined by W, Λ, the state matrix A, and a transformation matrix.

In general, there exist multiple real and complex solutions to Eq. (8). However, because the G_i are required to be real to yield a physically meaningful solution, only the real solutions of Eq. (8) are sought. All possible real solutions can be determined using techniques described by Ref. 23. There are typically at most $m!/(m/2!)$ real solutions to Eq. (8).

With all possible real solutions of X in hand, it is required to determine G_0 and G_1. It can be shown that the product FG_0 and FG_1 are not explicitly dependent on the choice of G_0 or G_1, but is only dependent on X. Therefore, either G_0 (or G_1) can be chosen arbitrarily, as long as its inverse exists. Then, G_1 (or G_0) is calculated from the relationship $X = G_1^{-1}G_0$

Thus, for each real solution X, we obtain different adjusted stiffness and damping matrices. A rationale for choosing the "best" adjusted stiffness and damping matrices is required. Because Eq. (8) is only a necessary, but not sufficient condition for symmetry, some of the adjusted matrices are not symmetric, and therefore are immediately eliminated as candidate solutions. Additionally, some of the adjusted stiffness and damping matrices no longer have the same definiteness as the original stiffness and damping matrices and, thus, are also eliminated as candidate solutions. The remaining solutions can then be inspected to see which best provide information concerning the extent of damage.

B. Eigenvector Expansion

The dimension of the experimental eigenvectors are typically much less than that of the analytical eigenvectors due to practical testing limitations. One solution to this problem is to employ a model reduction technique such that the reduced dimension of the analytical model matches that of the experimental eigenvector.[24] However, using such approaches may cause difficulties when trying to detect damage. Therefore, the approach taken in this work is to expand the measured eigenvectors to the dimension of the analytical eigenvectors.

1. Optimal Least Squares Expansion

It is not always possible to assign exactly to the adjusted FEM the eigenvectors that were measured. Therefore, in the optimal least squares expansion technique, the best "achievable" eigenvectors are determined. The achievable $n \times 1$ eigenvectors $\boldsymbol{v}_{ia}$ of the system defined by Eq. (1) must lie in the subspace defined by

$$L_i = (M\lambda_i^2 + D\lambda_i + K)^{-1}B_0, \qquad i = 1, \ldots, p \tag{9}$$

where λ_i is the measured eigenvalue associated with the ith measured eigenvector $\tilde{\boldsymbol{v}}_i$. The vector $\tilde{\boldsymbol{v}}_i$ is a $n \times 1$ representation of the s components of the measured eigenvector. The s components are placed in $\tilde{\boldsymbol{v}}_i$ according to the finite element node number corresponding to the measured component location. All other elements of $\tilde{\boldsymbol{v}}_i$ (which represent unmeasured components) are set to zero. Define a transformation Q that reorders the eigenvector such that those positions of the analytical model that correspond to measured eigenvector locations now comprise the top portion of the vector

$$\boldsymbol{v}_i = Q\tilde{\boldsymbol{v}}_i = \begin{bmatrix} \boldsymbol{u}_i \\ \boldsymbol{d}_i \end{bmatrix} \tag{10}$$

The $\boldsymbol{u}_i$ would then be the measured $s \times 1$ eigenvector $\tilde{\boldsymbol{v}}_i$, and $\boldsymbol{d}_i$ is the $(n\text{-}s) \times 1$ vector of free entries. Applying the same transformation to Eq. (9), one can partition the subspace as

$$QL_i = \begin{bmatrix} \tilde{L}_i \\ D_i \end{bmatrix} \tag{11}$$

The best achievable eigenvector $\boldsymbol{v}_{ia}$ in a least square sense is then given by

$$\boldsymbol{v}_{ia} = L_i[\tilde{L}_i^*\tilde{L}_i]^{-1}\tilde{L}_i^*\boldsymbol{u}_i, \quad i = 1, \ldots, p \tag{12}$$

The calculation of the p achievable eigenvectors using Eq. (12) requires p inversions of a $n \times n$ matrix. Although the matrix to be inverted is typically banded, this may present a practical computational difficulty when dealing with large FEMs. This computational burden can be alleviated by posing the eigenvector expansion as an orthogonal Procrustes problem.

2. Orthogonal Procrustes Expansion

The orthogonal Procrustes (OP) problem can be viewed as a search for a linear relationship between the original analytical eigenvectors and the measured eigenvectors[25,26]

$$\overline{v}_{ia} = \begin{bmatrix} \overline{u}_m \\ \overline{d}_m \end{bmatrix} = \begin{bmatrix} \overline{u}_a \\ \overline{d}_a \end{bmatrix} P_{op} \tag{13}$$

where $\overline{u}_m$ is the matrix of experimentally measured eigenvectors, $\overline{u}_a$ is the matrix of analytical eigenvectors with components corresponding to the measured locations, $\overline{d}_m$ is the matrix of unmeasured experimental eigenvectors to be determined, $\overline{d}_a$ is the matrix of analytical eigenvectors with components corresponding to the unmeasured locations and P_{op} is the transformation matrix. The orthogonal Procrustes problem can then be formally stated as

$$\min_{\text{wrt } P_{op}} \|\overline{u}_m - \overline{u}_a P_{op}\|_F \quad \text{subject to } P_{op}^T P_{op} = I_p \tag{14}$$

The solution to Eq. (14) is given by

$$P_{op} = \Psi\Phi^T \tag{15}$$

where Ψ and Φ are the left and right singular matrices of the matrix Y defined by

$$Y = \overline{u}_a^T\overline{u}_m = \Psi\Sigma\Phi^T \tag{16}$$

The measured eigenvector components are then augmented with the analytical components transformed by the optimal rotation matrix

$$\bar{d}_m = \bar{d}_a P_{op} \tag{17}$$

Note that all the achievable eigenvectors can be determined in a single calculation. Additionally, the OP expansion requires just a single singular value decomposition of Y, a $p \times p$ matrix. This singular value decomposition requires approximately $13p^3$ flops. Practical experience in using the orthogonal Procrustes expansion for measured complex modes indicates that more accurate expansions are obtained when performed on a single mode at a time. That is, instead of solving Eq. (14) where the u's are matrices, solve Eq. (14) repeatedly for the case where the u's are vectors. This also results in a simplification of the solution because Y is now a complex scalar. For this case, the optimal rotation is given as

$$P_{op} = \frac{Y_i}{|Y_i|} \qquad i = 1, p \tag{18}$$

C. Selection of B_0—Subspace Rotation

At this point, the control influence matrix B_0 is a free variable. The only condition placed on B_0 is that it must be of full rank. Therefore, different selections for B_0 may result in different adjusted damping and stiffness matrices. Thus, a rationale for selecting B_0 is required. A previous technique involves selecting B_0 such that the unmeasured modes of the analytical model are nearly uncontrollable.[17] This selection technique fixes the subspace in which the achievable eigenvectors must lie. For model refinement, this may be valid because the difference between the original and measured eigenvectors may be small. However, this may not be the case when damage is present. Thus, a new selection technique is developed that defines the subspace such that the measured eigenvectors lie nearly in the subspace.

The subspace rotation algorithm is based on selecting B_0 such that the measured eigenvector lies in the achievable eigenvector subspace defined by Eq. (9). In this method, the measured eigenvalues and eigenvectors are assigned to the analytical model individually. Thus, each measured mode will have its own B_0. The control influence matrix B_0 for the ith mode is given as

$$B_0 = Re\{(M\lambda_i^2 + D\lambda_i + K)L_i\} \tag{19}$$

where the first column of L_i is the measured eigenvector and the second column is arbitrary. Essentially, Eq. (9) has been solved for B_0. Note that using the orthogonal Procrustes expansion technique in conjunction with subspace rotation eliminates the need of inverting an $n \times n$ matrix.

IV. Load Path Preservation

At this point, additional load paths may have been introduced in the updated stiffness and damping matrices; that is, elements of the stiffness and damping matrices that were originally zero may now be nonzero. Whether this is a practical problem is still a matter of current debate but it seems that for damage detection of truss structures it is desired to maintain load paths. The original load path definition can be enforced in the eigenstructure assignment technique using an iterative approach. Denote K_m and D_m as stiffness and damping masking matrices, respectively, that are defined on an element-by-element basis as:

$$\begin{aligned} K_m(i,j) &= 1 \text{ if } K(i,j) \neq 0 \\ D_m(i,j) &= 1 \text{ if } D(i,j) \neq 0 \\ K_m(i,j) &= 0 \text{ if } K(i,j) = 0 \\ D_m(i,j) &= 0 \text{ if } D(i,j) = 0 \end{aligned} \tag{20}$$

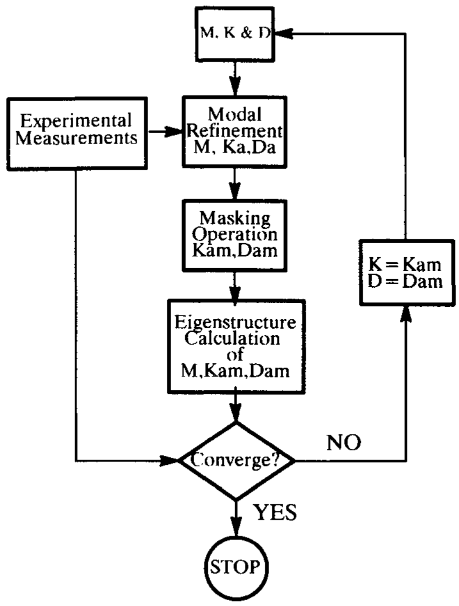


Fig. 1 Iterative load path preservation.

Defining the element-by-element (scalar) matrix multiplication operator as:

$$A = B \odot C \quad A(i,j) = B(i,j)^*C(i,j) \quad i, j = 1, n \tag{21}$$

the adjusted and masked stiffness and damping matrices are given as:

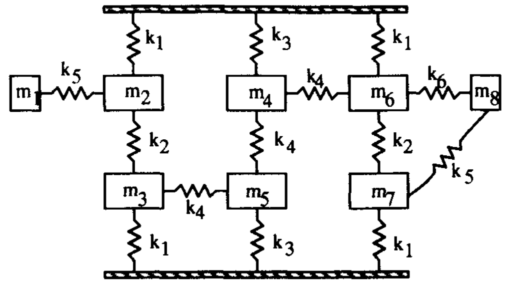
$$K_{a,m} = K_a \odot K_m \quad D_{a,m} = D_a \odot D_m \tag{22}$$

The adjusted and masked analytical eigenstructure will no longer match the experimentally measured eigenstructure, although typically it will be much closer than that of the original analytical model. Therefore, the entire algorithm can be repeated using the adjusted and masked stiffness and damping matrices in place of the original stiffness and damping matrices, as shown in Fig. 1. This iterative process can be repeated until the difference in the eigenstructure of the adjusted (K_a and D_a) and the adjusted and masked ($K_{a,m}$ and $D_{a,m}$) is resolved to within either user set limits or computational machine precision. It should be noted that there is no formal guarantee of convergence in using this algorithm. Experience indicates that if the experimental data is consistent with the sparsity pattern, the algorithm will converge. By consistent data, we mean that there exists a stiffness and damping matrix that has as a subset of its eigenstructure the measured test data *and* that also exhibits the same sparsity pattern as the original matrices. If the data is inconsistent, the sparsity pattern will not be preserved, but in general the nonzero terms of K_a and D_a, which should be zero, will be closer to zero after application of the algorithm. The selection of which X real solution of Eq. (8) to use at each iteration step is guided by which solution yields the greatest insight on the location and extent of damage.

V. Numerical Illustrations

A. Introduction

To illustrate characteristics of the damage detection algorithm, two example problems are presented. The first problem is a widely used spring-mass example.[6] It is used here for the purposes of illustrating model refinement for a large local discrepancy, analogous to a damage detection situation. The phenomena of global/local mode switching and load path preservation is examined in this problem. The second problem involves the determination of damage in a model of a laboratory cantilever beam using measured modal parameters.



$m_1 = 0.001$ $m_8 = 0.002$ $m_j = 1.0$ $j=2,7$

$k_1 = 1000$ $k_2 = 10$ $k_3 = 900$ $k_4 = 100$ $k_5 = 1.5$ $k_6 = 2.0$

Fig. 2 Kabe's problem.

B. Damage Detection—Kabe's Problem

Kabe's eight degree of freedom spring-mass problem is shown in Fig. 2, which includes the stiffness and mass values for the exact model. This problem presents a challenging situation for damage detection in that stiffness values of various magnitudes are included. The model exhibits closely spaced frequencies and exhibits both local and global modes of vibration.

A variation of Kabe's original problem is used here. Rather than the standard initial model commonly used, which has incorrect values for all of the connecting springs, only a single spring constant is changed. This is reflective of the fact that damage may occur as a large local change in the stiffness of a structural member.

1. Local-to-Global Mode Change

In the first problem, the initial model is only incorrect for the spring between masses 3 and 5. A value of 500, five times that of the exact spring, is assumed in this problem. Changing the spring value from 500 to 100 also causes a local mode of vibration to be replaced by a global mode, thus presenting a difficult challenge for damage detection.

Figure 3 presents element-by-element stiffness matrix results for applying Baruch's update and the eigenstructure assignment algorithm. Baruch damage indicates that the update was made using Baruch's algorithm. SEA-M damage indicates that the update was made using the eigenstructure assignment algorithm with B_0 selected using the modal (M) selection method. SEA-SR damage indicates that the update was made using the eigenstructure assignment algorithm with B_0 selected using Subspace Rotation (SR). Because in all examples each mode is assigned independent of all other modes, there are typically two real solutions to the algebraic Riccati equation. The x-axis on all plots are the indices of a column vector constructed by storing the upper triangular portion of the stiffness matrix in a column vector. The y axis on all plots consist of the difference between the updated stiffness matrix elements and the original stiffness matrix.

In the first case, as shown in Fig. 3, it is assumed that only the fundamental mode of vibration is measured, *but* all eigenvector components have been measured. Thus, no expansion of eigenvectors is required. It is evident from Fig. 3 that the Baruch update is unable to discern damage, but that both the SEA-M and SEA-SR is able to clearly locate damage. In fact, the SEA-SR was able to exactly reproduce the correct stiffness matrix. This was true independent of which mode was used in the update. However, it should be noted that there is no guarantee that this will hold true in general. It should be noted that Baruchs update tends to focus elemental changes in the third and fifth row of the stiffness matrix, indicating the possibility of damage between these degrees of freedom, but certainly giving no clear indication to the extent of damage. As is evident from the plot, the Baruch update has spread errors over several elements. Using the algorithm of Ref. 19, the damage vector is given as $\alpha = [1.0\ 0.93\ 0.72\ 0.83\ 0.70\ 0.90\ 0.97\ 1.0]^T$, where the element number corresponds to the structural DOFs and a number less than 1 indicates the possibility of damage affecting that DOF. It is obvious that DOFs 3 and 5 are affected by damage, but the results also indicate strong damage of DOF 4.

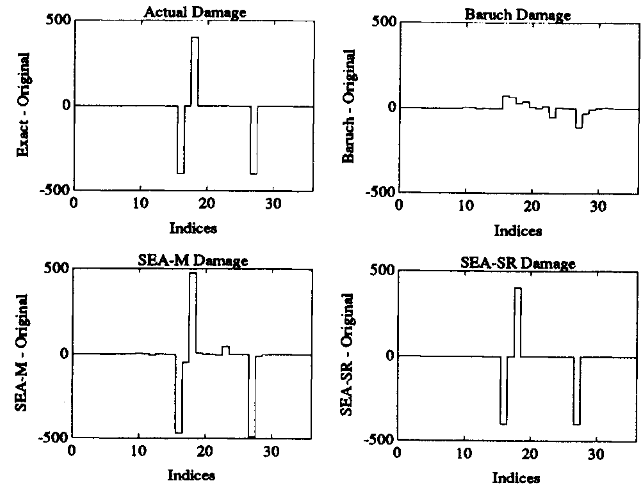


Fig. 3 First mode, full eigenvector.

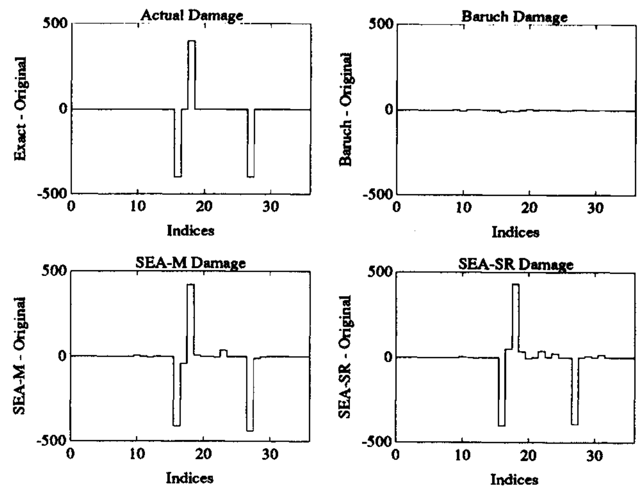


Fig. 4 Modes 1, 2, 3—eigenvector components.

In the second case, as shown in Fig. 4, it is assumed that the first three modes of vibration have been measured, *but* only the first three components of the eigenvectors have been measured. The eigenvector components were expanded for the Baruch update using dynamic expansion[5] with subsequent orthogonalization.[4] The least-squares expansion was used for the SEA-M update. The SEA-SR update utilized the orthogonal Procrustes expansion. In comparing Fig. 4 to Fig. 3, it is clear that the damage detection capability of all three algorithms have been degraded when using expanded mode shapes, even though more modes have been measured. However, both the SEA-M and SEA-SR updates give a clear indication to both the location and extent of damage. Using the algorithm of Ref. 19, the damage vector is given as $\alpha = [1.0\ 0.81\ 0.75\ 0.83\ 0.82\ 0.79\ 0.85\ 1.0]^T$. It is difficult from inspection of α to determine the location of damage.

The effect of applying the iterative load path algorithm in the update procedure is shown in Fig. 5. For the Baruch update, 100 iterations were performed. For the SEA-M and SEA-SR updates, two and three iterations respectively were performed. The iterations were halted early for both SEA updates because the discrepancy between the eigenstructure before and after masking was within the numerical precision of the eigenstructure assignment software. It is seen that the load path enforcement further enhances the damage detection capability of the SEA updates.

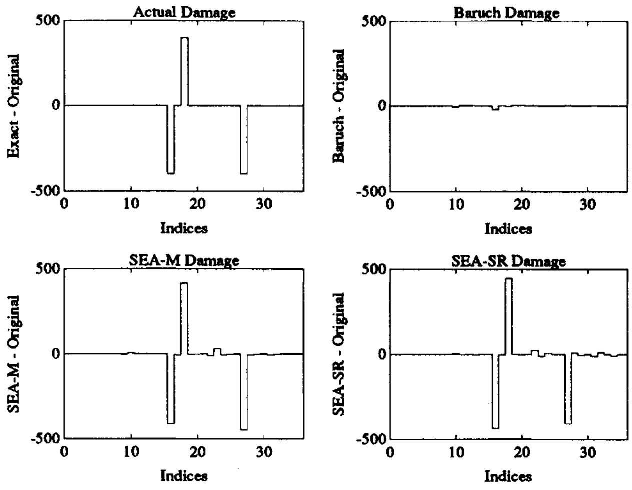


Fig. 5 Load path preservation.

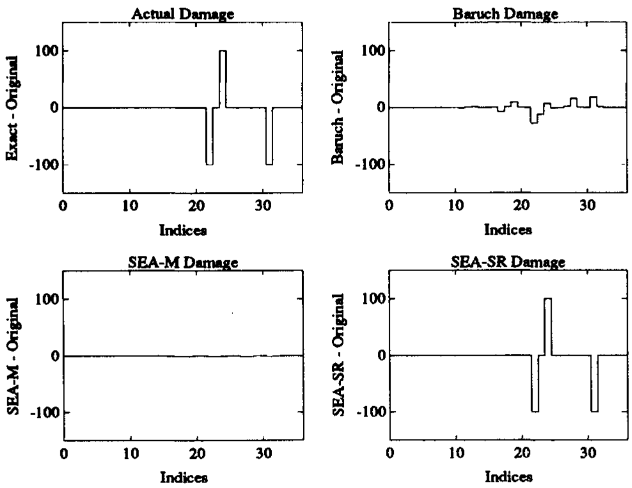


Fig. 6 First mode, full eigenvector.

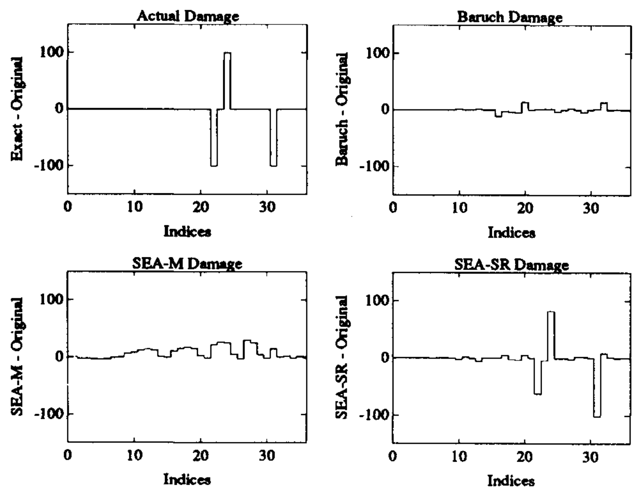


Fig. 7 Mode 1, 2, 3—eigenvector component 1, 3, 6.

2. *Consistent Modes*

In the second problem, the initial model is only incorrect for the spring between masses 4 and 6. A value of 200, two times that of the exact spring, is assumed in this problem. In this problem, all global and local modes remain global and local modes respectively after damage. It should be noted that finding a problem with this feature was difficult.

In the first case, as shown in Fig. 6, it is assumed that only the fundamental mode of vibration is measured, *but* all eigenvector components have been measured. It is evident from Fig. 6 that the Baruch and SEA-M update in unable to discern damage, but that the SEA-SR is able to locate damage clearly. In fact, the SEA-SR was able to reproduce exactly the correct stiffness matrix. Again, this was true independent of which mode was used in the update. It should be noted that the Baruch and SEA-M update tends to focus elemental changes in the fourth and sixth rows of the stiffness matrix, indicating the possibility of damage between these degrees of freedom, but certainly giving no clear indication to the extent of damage. Using the algorithm of Ref. 19, the damage vector is given as $\alpha = [1.0\ 0.98\ 0.92\ 0.85\ 0.87\ 0.84\ 0.95\ 1.0]^T$. This algorithm does not identify the damage location clearly.

In the second case, as shown in Fig. 7, it is assumed that the first three modes of vibration have been measured, *but* only DOFs 1,3, and 6 of the eigenvectors have been measured. In comparing Fig. 7 to Fig. 6, it is clear that the damage detection capability of all three algorithms has again been degraded when using expanded mode shapes. Only the SEA-SR update gives a clear indication to the location of damage, but is unable to predict the exact extent. Using the algorithm of Ref. 19, the damage vector is given as $\alpha = [0.99\ 0.18\ 0.55\ 0.34\ 0.52\ 0.33\ 0.41\ 1.0]^T$. It is difficult from inspection of α to determine the location of damage. In fact, inspection of α indicates that DOF 2 is the most likely damaged DOF.

It should be noted that in this problem, it was critical to have the proper DOFs measured. When the second test case was run with having the first three DOFs measured, no algorithm was able to even locate damage. In this case, the eigenvectors components were relatively unaffected by damage, thus causing substantial error in the eigenvector expansion process.

C. Experimental Study

1. *Modal Test Description*

The tested structure was a cantilevered beam, as shown in Fig. 8. The dimensions and properties of the structure are given in Table 1. Six equally spaced translational degrees of freedom were selected as measurement locations as indicated. The modal properties of the first three modes of vibration were determined using frequency domain techniques and single degree-of-freedom curve-fitting algorithms. The excitation source used was an impact hammer and the driving point measurement was at the free end of the beam. Impact and exponential windows were utilized to improve frequency response calculations. At each measured degree of freedom, five frequency response measurements were made and averaged to reduce the effects of measurement noise. The results of the modal test are listed in Tables 2 and 3.

2. *Finite Element Model Description*

A simple FEM was constructed using beam elements. The beam element has two degrees of freedom at each node;

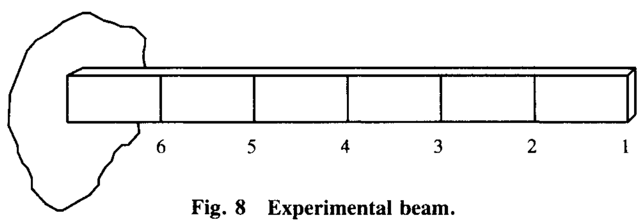


Fig. 8 Experimental beam.

Table 1 Structural properties

Length—0.84 m
Mass/length—2.364 kg/m
Moment of inertia—3.02e-9 m^4
Youngs modulus—70 GPa

Table 2 Measured natural frequencies and damping ratios

Mode #	Natural frequency, Hz	Damping ratio, %
1	7.27	4.41
2	45.55	0.68
3	127.01	0.33

Table 3 Measured mode shapes

DOF	Mode 1	Mode 2	Mode 3
1	1.00	1.00	1.00
2	0.95	0.16	−0.30
3	0.65	−0.53	−0.61
4	0.36	−0.72	0.20
5	0.15	−0.52	0.75
6	0.03	−0.12	0.28

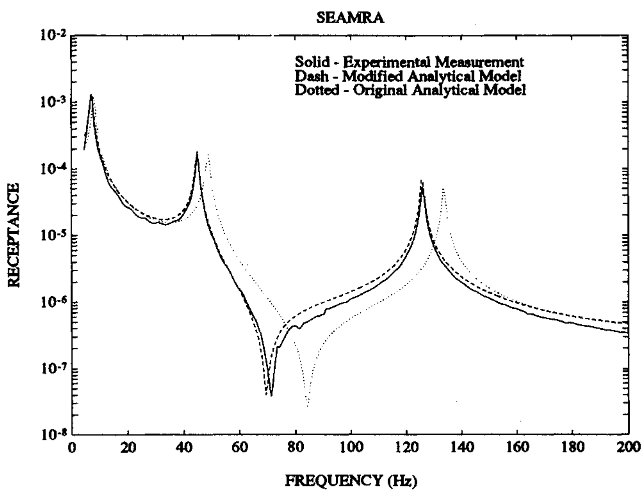


Fig. 9 Experimental/analytical frequency response function.

bending and rotation. The initial FEM had six elements. This model was then reduced using Guyan reduction, eliminating the rotational degrees of freedom. There are several possible errors introduced into the FEM, most obvious being the fact that a perfect cantilever condition is assumed. In addition, an artificial error of selecting the Young's Modulus higher than that often assumed for aluminum will contribute to the error in the original finite element model. Thus, the damage takes the form of having a global reduction in the Young's Modulus, and that a true cantilever boundary condition is not achieved in the laboratory setup.

3. Application of Damage Detection Algorithm

Because the "true" finite element model is unknown, one cannot make comparisons between the "true" and updated structural matrices. Besides a comparison of measured modal properties, a useful judge on the quality of the damage detection capability can be obtained by comparing actual experimental frequency response functions with those predicted by the FEM. Figure 9 shows a comparison of frequency response functions measured between degrees of freedom 1 and 3. The solid curve corresponds to the experimental data, while the dotted line corresponds to that predicted by the original analytical FEM. It is apparent that the discrepancies between the frequency response function increases as the frequency of excitation increases. This is in part due to the fact that the assumption of a perfect cantilever condition affects the higher modes of vibration to a greater extent. The dashed line in this figure corresponds to the SEA-SR updated finite element model. Inspection of the updated stiffness matrix indicates that changes occur throughout the matrix, indicating that the damage was due to degradation of some global structural property (Young's Modulus), as opposed to some form of local damage.

VI. Summary

In this work, an eigenstructure assignment technique has been tested for its suitability as a structural damage location algorithm. The algorithm makes use of a pre-damage refined finite element model of the structure and the results of a post-damage modal survey. The algorithm determines perturbation matrices to the original finite element model such that the updated model exhibits the measured modal characteristics of the damaged structure. The perturbation matrices are then examined to determine, if possible, the location and extent of structural damage.

A technique to enhance eigenvector assignability has been developed. The method is based on rotating the achievable eigenvector subspace into the experimentally measured and possibly expanded eigenvector. The subspace rotation method, used in conjunction with an orthogonal Procrustes expansion technique results in both a decrease in the computational burden, as well as an increase in the accuracy of the assigned eigenvector.

An iterative algorithm has been developed to enforce the load path constraints set forth by the original finite element model. Experience gained in using the algorithm indicates that convergence is usually obtained if the measured modal data are consistent with the original load path constraint. Inconsistent data result in the algorithm stalling; that is, additional load paths are introduced in the updated model.

Finally, the algorithm was tested on both "simulated" and actual experimental data. A challenging simulated structure was used as a test bed. The structure exhibits closely spaced frequencies and has both global and local modes of vibrations. The algorithm performed well on the test structure, although experience indicates that the accuracy of results is dependent on which degrees of freedom are represented in the measured mode shapes. The algorithm was also tested using the results of an experimental modal test of a cantilevered beam. A comparison of measured and analytical frequency response functions indicated the improvement made by the eigenstructure assignment algorithm.

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